A General Equilibrium perspective on Offshoring and Economic Growth

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In this paper we show that differences in factor endowments underlie firms' decision to go offshore and we study some of the implications for the host country. Using an endogenous growth model we find that offshoring impacts factor prices and returns, while boosting the growth rate of the host economy by stimulating human and physical capital accumulation. Results show that offshoring may initially be welfare-reducing, but also point to the fact that it subsequently fosters consumption growth and alters the factor content of exports. The same feature, which is typical of the global economy era, bears important implications for the host country's current account position.

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1. Introduction

Since the seminal work of David Ricardo in 1817 the nature of international trade has undergone significant changes. From the comparative advantage explanation of trade flows, international trade now involves goods whose production process is split across borders.²

Profit motivations are generally said to be driving this constant evolution, and throughout time the economic literature has identified many underlying triggers, including reductions in transportation costs, the opportunity to tap into local resources, access to low-cost inputs, attempts to bypass the tariffs that protect a market from imported goods, etc.

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² Many studies have now documented the global disintegration of the production process (see, e.g., Yeats, 2001; Hummels, Jun Ishii, and Yi, 2001; Hanson, Mataloni, and Slaughter, 2001 and 2005)

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In line with this new reality of international trade, a recent and fast growing literature studies the micro and macroeconomics of offshoring, i.e. the global disintegration of the production process. This paper aims at contributing to this literature by shedding light on the macroeconomic outcomes of offshoring as experienced by a host country. We develop a general equilibrium model of endogenous growth to investigate the impacts of offshoring on factor prices and returns, the growth rate of the economy, the factor content of exports, and the current account position. Our analysis is consistent with a structural transformation of the host country as we find that, among other things, offshoring fosters human and physical capital accumulation while altering the factor content of exports. Results derive from the firm's cost-saving calculations which map offshoring into a factor-augmenting technological progress in the host country.3

Our model features the new reality of North-South trade relations and assumes two trading nations, one of which displays more human and physical capital and which we refer to as "North". As the other nation, "South", displays less of both factors, we first show that these differences in factor endowments give rise to a price-wedge for intermediate goods unfavorable to the former nation.

In the current framework, two final goods - a consumption good and human capital - obtain by packaging together a continuum of differentiated intermediate goods. Firms may carry out the packaging task at home or abroad in order to supply the developing country's market. As profit-maximization requires that firms meet demand at the lowest cost, they factor in both international transportation costs and the price-wedge for intermediate goods between the two economies. Whether the tasks performed offshore are within or beyond the boundaries of the firm is of little interest for the current analysis. To keep matters simple we assume that a firm needs the same amount of a given intermediate good no matter if the production process is concentrated at home, vertically integrated through a foreign subsidiary or outsourced to a foreign supplier. For the host country, the extent of extra demand for intermediate goods generated by offshore activities is of prime importance, especially as backward and forward linkages generally ensure that soaring activities in one sector spill over to other sectors.4

With perfectly competitive markets for final goods, and provided that the price-wedge for intermediate goods falls short of transportation costs, the only viable option to compete in the developing country's market is to produce final goods locally using locally-acquired intermediate

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4 Some industries clearly display higher value added and thus, should ideally be the ones targeted by offshoring foreign firms. However, we elected to keep the analysis simple enough to highlight the general principles involved in a growth episode driven by offshore activities.
goods. As a result, demand for intermediate goods in the developing country increases in proportion to the price gap between both economies. Clearly, the higher the price differential between intermediate goods from the two economies, the larger North-South offshoring flows. Whether the final product is also shipped North or not, clearly depends on whether the price-wedge is worth transportation costs between the two economies. We first rule out that possibility for both final and intermediate goods, and we shed light on the fundamental mechanism at play. We subsequently assume that the price-wedge over final goods is worth transportation costs and some interesting implications for the host country's foreign account position follow.

The contribution of this study is two-fold. On the one hand, it connects to the literature on offshoring, and on the other hand, it addresses the issue of structural transformation mechanisms in developing countries.

Over the last decade many contributions have been made to the literature on offshoring and most of those focus on a firm's choice of an organizational form.\(^5\)

In that strand of the literature the emphasis is on imperfect information issues, matching problems or sophisticated contracting schemes to rationalize the firm's production arrangements. In our model, we keep the issue of a firm's choice of a production arrangement at minimum by showing that the decision to geographically separate management and production activities is driven by cost-minimization considerations, and we focus attention on downstream implications for the host country.

Another strand of the literature on offshoring consists of models of "fragmentation" - the breakdown of technology for producing some good into separate parts that can be carried out in different places.\(^6\) Grossman and Rossi-Hansberg (2008) recently synthesized the general principles of this second strand by developing a model of offshoring as a continuous and ubiquitous phenomenon. They investigate how falling costs of offshoring affect factor prices in source country, and they identify a productivity effect, a relative-price effect, and a labor-supply effect. These authors show that improvements in the technology for offshoring tasks amount to a labor-augmenting technological progress that improves the real wages in the source country. Egger and Falkinger (2003) also develop a general equilibrium model to examine the assumptions under which international offshoring can be treated as technical progress in the source country. In their analysis of the distributional effects they distinguish two driving forces, namely, a substitution and a cost-saving effect. Our analysis complements Grossman and Rossi-

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Hansberg (2008) by showing that offshoring also amounts to a factor-augmenting technological progress affecting both capital and labor in the host country. It also complements Egger and Falkinger (2003) by featuring a general equilibrium model in which the firm's cost-saving strategy bears growth and welfare consequences for the host country. Both Grossman and Rossi-Hansberg (2008), and Egger and Falkinger (2003) consider the distributional effects of offshoring from the source country's perspective, whereas our focus is on growth and welfare in the host country. Altogether, Grossman and Rossi-Hansberg's (2008) findings and ours replicate the well-known result of the international trade theory according to which there are mutual gains from trade for all trade partners.

The paper also adds to the literature on structural transformations in a significant way. The growing tendency for developing countries to export skill and capital intensive goods contrast significantly with the standard Heckscher-Ohlin predictions of international trade patterns. This mainstream theory predicts that labor-intensive developing countries should have a comparative advantage in primary goods and unskilled-labor, rather than skill and technology, intensive activities. Recent contributions to this literature emphasize sectoral differences in total factor productivity growth in explaining structural transformations, the shifts in industrial employment shares over long periods of time.7

Unlike these models, ours emphasizes waves of offshoring as a potential triggering factor for structural transformation.8

In our framework, differences in factor endowments explain differences in production costs, which in turn motivate firms' decisions to go offshore. By increasing the demand for intermediate goods, offshoring impacts factor prices and returns in the host country, and triggers structural transformation through direct human and physical capital accumulation.

In fact, for a developing country with a price-advantage over intermediate goods

1. Offshoring raises factor prices, and even more so, factors return;
2. Offshoring fosters direct human and physical capital accumulation, and as such, accelerates economic growth;
3. Offshoring raises the (physical) capital intensity of tradeables; and,
4. While being initially welfare-reducing, it spurs consumption growth thereafter;

8 See, e.g., Ngai and Pissarides (2007).
5. Offshoring improves the developing country's net foreign asset position in the steady-state of the economy.

The paper is structured as follows. Section II outlines the model. In both countries, monopolistically competitive firms produce intermediate goods using a standard Cobb-Douglas technology. In the final good sectors, perfectly competitive firms carry out product assembly using a standard CES technology to produce a final consumption good (our *numeraire*), and human capital. We first show how differing factor endowments create a "North" - "South" price-wedge for intermediate goods. This feature is then mapped into a generic index describing the scope of offshoring flows to the developing nation. The consumption side of the model consists of a large number of homogenous households, and the representative household in each country is endowed with homothetic preferences defined over the final consumption good only. Following the lead of Echevarria (2008), the representative consumer owns the factors of production in each country and also decides their distribution between alternative uses. In Section III we investigate a first set of implications for the developing economy assuming that locally-produced final goods only serve local markets. To the extent that transportation costs outweigh the price-wedge between both economies, intermediate goods are internationally non-traded as well. Section IV proceeds under the assumption that the price-wedge for final goods is worth transportation costs and an analysis of the developing nation's current account position follows. Section V contains concluding remarks.

2. The Model

The model builds on Lucas (1988) and Romer (1990). We consider an environment in which economic activities extend over an infinite number of periods $t = 0, 1, ...$. Two nations coexist in this environment and in both economies there exists a continuum of intermediate goods, including human and physical capital. Intermediate goods can be packaged separately to get a composite human capital good on the one hand and a composite physical good on the other hand. While the latter is tradeable, human capital is non-tradeable.

At the beginning of the economy the developed nation displays more of both composite goods. We maintain this assumption throughout the analysis regardless of any addition that may occur in either of both countries. As we show in the next sub-section, this distribution of factor endowments implies that intermediate goods are typically cheaper if purchased from the

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9 For instance, such various fields as history, mathematics, biology, physics and economics all contribute to the instruction of, say, a civil engineer.

10 This assumption is consistent with the fact that emerging and developing countries' capital per worker and average human capital still fall short of those of developed nations, although the gap may be shrinking in some instances.
developing country. However, moving goods across borders involves a transportation cost of $c$ per unit. Thus, should the price-wedge for intermediate goods outweigh transportation costs, the final consumption good will be produced worldwide using intermediate goods from the developing country only. Otherwise, the developing country's cost advantage makes it appealing for firms from the developed nation to produce final goods South using locally-produced intermediate goods. Since the first alternative stands at odds with real world facts, we shall investigate the latter case, especially as it captures the prospects for offshore activities.

In either country the representative household owns factors of production which it rents out to firms. Thus, a firm engaging in offshoring needs not transfer production resources internationally. In fact, like its local competitors it may rent out capital from the local owner, i.e. the local representative household.

2.1. The Production of Final Goods

In this subsection we rationalize that differences in factor endowments between the two countries give rise to a price-wedge for intermediate goods, and this feature in turn underlies the prospects for offshoring.

In either country perfectly competitive firms produce the final physical good $Y_t$ according to a CES technology. Production consists of packaging together the intermediate physical goods, hence

$$Y_t = \left( \int_0^1 Y_t^{\theta} dj \right)^{1/\theta}, \quad 0 < \theta < 1$$

where $Y_t$ denotes total output, $Y_t$ the quantity of the tangible intermediate good $j$ used, and $\theta$ a positive parameter which captures the degree of substitutability between the different intermediate goods and as such, the market power of the corresponding suppliers, with $j \in [0, 1]$.

Since the tangible final good is either consumed or accumulated as next period stock of physical capital, prior to any exchange between the two nations the following identity holds in either country:

$$Y_t = C_t + I_t$$

where $C_t$ is aggregate consumption and $I_t$ total (physical) capital investment.

Next, since $Y_t$ is our numeraire, letting $P_{ij}$ denote the relative price of intermediate good $j$, profit maximization yields the following demand schedule for type-$j$ physical intermediate good:
Likewise, in both countries human capital composite \( X_t \) derives from a CES aggregation over human capital intermediate goods according to the following technology:

\[
X_t = \left( \int_0^1 X_{it}^\theta \, di \right)^{1/\theta}, \quad 0 < \theta < 1, \tag{4}
\]

where \( X_{it} \) denotes type-\( i \) human capital "good" with \( i \in [0, 1] \). Letting \( \rho_{it} \) denote its relative price, profit maximization in this sector yields the following demand schedule:

\[
X_{it} = \left( \rho_{it} / \rho_t \right)^{-1/(1-\theta)} X_t, \tag{5}
\]

where \( \rho_t \) is the price index associated with (4).

Equations (3) and (5) are the standard downward slopping demand curves facing the monopolists and underlie our first lemma.

**Lemma 1:** To the extent that the developed nation displays more of both human and physical capital than the developing one, intermediate goods are cheaper in the latter one.

**Proof:** We first use equation (3) to express \( P_{it} \) as a function of aggregate demand, \( Y_t \)

\[
P_{it} = \left( \frac{Y_t}{Y_{jt}} \right)^{(1-\theta)}. \tag{6}
\]

Prior to offshoring, it can be shown that \( Y_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] A(K_t)^\gamma (H_t)^{1-\gamma} \) (see step 1 toward proving Proposition 1 in the Appendix section).\(^{11}\) Hence, for each country, the counterpart to equation (6) is given by

\[
P_{it}^N = \left[ [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] A(K_t^N)^\gamma (H_t^N)^{1-\gamma} / Y_{jt}^N \right]^{(1-\theta)},
\]

\[
P_{it}^S = \left[ [1 - (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] A(K_t^S)^\gamma (H_t^S)^{1-\gamma} / Y_{jt}^S \right]^{(1-\theta)}.
\]

For a given need, \( Y_{jt}^N = Y_{jt}^S \), of intermediate good \( j \), it is easy to see that \( P_{it}^N > P_{it}^S \) as long as \( K_t^N > K_t^S \) and \( H_t^N > H_t^S \).

\(^{11}\) 0 < \( \varepsilon < 1 \) denotes the generic index of offshoring flows between both countries.
Similarly, using equation (5) we have that 
\[ \left( \frac{\rho_{it}^N}{\rho_t^N} \right) = \left( \frac{X_t^N}{X_{it}} \right)^{(1-\theta)} \] and 
\[ \left( \frac{\rho_{it}^s}{\rho_t^s} \right) = \left( \frac{X_t^s}{X_{it}} \right)^{(1-\theta)} \]. It then follows that human capital intermediate goods, \( X_{it} \), are cheaper, i.e.
\[ \left( \frac{\rho_{it}^N}{\rho_t^N} \right) > \left( \frac{\rho_{it}^s}{\rho_t^s} \right) \] in the developing country provided that \( X_t^N > X_t^S \), i.e. \( H_t^N > H_t^S \).

This ends the proof.

In an environment where both economies share the same technology and intermediate goods are cheaper in the developing country, it must be that the price-wedge for intermediate goods falls short of international transportation costs. Otherwise, firms could profitably assemble the consumption good North using intermediate goods from the less developed nation. This in turn would suggest that only intermediate goods from developing countries are used worldwide, a prediction that contrasts with the available evidence. Hence, letting \( \lambda \) denote the price-wedge over intermediate goods between the two countries, i.e. \( P_{jt}^N = P_{jt}^N + \lambda \), throughout the paper we shall assume that \( 0 < \lambda < c \).

In order to shed light on the potential growth effects of offshoring from the host country's perspective, we adopt the following approach for our analysis. We first assume that the price difference for the final consumption good is not worth transportation costs and we investigate the developing country's path of growth. This assumption mainly implies that packaging activities take place in both countries, although they might be on the rise in the developing economy given its cost advantage. Later on we investigate the case where the consumption good can be profitably traded internationally as we study the less developed nation's current account position.

### 2.2. The Production of Intermediate Goods

In both countries monopolistically competitive firms produce differentiated intermediate goods indexed by \( j \) (for physical intermediate goods) and \( i \) (for intermediate "human capital goods"), with \( i, j \in [0,1] \). When producing intermediate goods, firms combine human and physical capital composites according to the following Cobb Douglas production function:

\[ y_{jt} = A(K_{jt})^{\gamma}(H_{jt})^{1-\gamma}, \quad 0 < \gamma < 1 \quad (7) \]
\[ X_{lt} = Z(K_{lt})^\gamma (H_{lt})^{1-\gamma}. \]  

\( K_{ht} \) (respectively, \( H_{ht} \)) is physical capital (respectively, human capital) rented out by firm \( h = i, j \), \( A \) and \( Z \) are positive technological parameters which we assume time-invariant. To keep matters simple we also assume that these technological parameters are the same for both countries.

Next, in either country human and physical capital composites can each be accumulated for next period. However, they also fully depreciate after usage, hence the following laws of motion:

\[ K_{t+1} = I_t, \]
\[ H_{t+1} = X_t, \]

Economy-wide, in each country firms use \( K_{yt} \) (respectively, \( H_{yt} \)) units of physical capital (respectively, human capital) to produce intermediate physical goods. Thus, each country's market clearing conditions for either type of good are as follows:

\[ K_t = K_{xt} + K_{yt}, \quad H_t = H_{xt} + H_{yt} \]  

with

\[ K_{xt} = \int_0^1 K_{lt} \, di, \quad K_{yt} = \int_0^1 K_{jt} \, dj, \quad H_{xt} = \int_0^1 H_{lt} \, di, \quad \text{and} \quad H_{yt} = \int_0^1 H_{jt} \, dj \]  

2.3. Preferences

In either country a large number of homogenous households have preferences defined over expected streams of consumption \( C_t \). The representative household ranks alternative streams of consumption using the following expected utility function:

\[ E_t \sum_0^\infty \beta^t \ln C_t, \]  

where \( 0 < \beta < 1 \) denotes the usual rate of time discounting, and the operator \( E_t \) the mathematical expectation conditional on period - \( t \) information. Under the working assumption that the price-wedge is not worth transportation costs, \( C_t \) is produced locally.

The representative household seeks to maximize (13) subject to the sequence of its periodic budget constraints, and time - \( t \) budget constraint is given by:

\[ C_t + K_{t+1} + \rho_t H_{t+1} = R_t K_t + W_t H_t + \Pi_t, \]
where $K_{t+1}$ denotes investment, $K_t$ and $H_t$ stand for the accumulated stocks of physical and human capital respectively, $\rho_t$ is the relative price of human capital in terms of the consumption good, $R_t$ is the monetary return on physical capital, $W_t$ is the nominal wage rate, and $\Pi_t$, the nominal profits accruing to the household. Equation (14) illustrates the facts that (i) the tangible final good is the numeraire, (ii) capital fully depreciates after usage, and (iii) the representative household receives the profits generated by monopolistic competition in the intermediate sector. Solving for the optimal values of $C_t, H_{t+1}$ and $K_{t+1}$, it can be shown that

\[
\frac{\rho_t}{C_t} = \beta E_t \left( \frac{W_{t+1}}{C_{t+1}} \right),
\]

and,

\[
\frac{1}{C_t} = \beta E_t \left( \frac{R_{t+1}}{C_{t+1}} \right).
\]

Combining equations (15) and (16) unveils a one-to-one relationship between the current relative price of the composite human capital good and its expected relative return:

\[
\rho_t = E_t \left( \frac{W_{t+1}}{R_{t+1}} \right).
\]

On the other hand, since the consumption good is our numeraire, it follows that aggregate demand amounts to $Y_t + \rho_t X_t$ in value terms. Thus, in value terms, the aggregate supply - aggregate demand equilibrium, $\psi_t = Y_t + \rho_t X_t$, implies the following constraint on resources:

\[
\psi_t = C_t + K_{t+1} + \rho_t X_t,
\]

where, according to our specifications in (1), (4), (7), (8) and (12), national output in either country is expressed as:

\[
\psi_t = B(K_t)^\gamma (H_t)^{1-\gamma},
\]

with aggregate total factor productivity, $B$, being some combination of sectoral TFP, $A$ and $Z$ in each country.

Equations (17) - (19) prove useful in the next section as we turn to investigating changes to the developing country's economy in the light of offshore activities from abroad.
3. Factor Price Differentials and Offshoring: Implications for the Host Country

Under the maintained assumption that transportation costs outweigh the price-wedge between both economies, i.e. $0 < \lambda < c$, demand for intermediate goods in the developing nation increases proportionally to the price difference between North and South. This is because $0 < \lambda < c$ implies that firms cannot profitably import intermediate goods from the developing nation, nor can they profitably export the consumption good South. Thus, Northern firms willing to compete in the developing nation's market must carry out product assembly locally, using locally-produced intermediate goods.

Let $F(.)$ define a function such that $\varepsilon = F(\lambda)$, where $0 < \varepsilon < 1$ denotes the generic index of offshoring flows associated with a price-wedge $\lambda$. The larger the price-wedge, the larger offshoring flows to the developing nation and the higher the index. Since offshore production activities consist of packaging intermediate goods abroad, the extent to which this feature alters the demand for locally-produced intermediate goods necessarily depends on the scope of offshoring flows.

Thus, using equations (5) and (3), we can define the altered demand schedules as follows:

\[
\begin{align*}
\tilde{Y}_{jt} &\equiv (1 - \varepsilon)^{-1}Y_{jt} = (1 - \varepsilon)^{-1}p_{jt}^{-1/(1-\theta)}Y_t, \quad (20) \\
\tilde{X}_{lt} &\equiv (1 - \varepsilon)^{-1}X_{lt} = (1 - \varepsilon)^{-1}(p_{lt}/\rho_{lt})^{-1/(1-\theta)}X_t. \quad (21)
\end{align*}
\]

Given the initial aggregate demands for human and physical capital, $Y_t$ and $X_t$, and provided that $0 < \lambda < c$, equations (20) and (21) are consistent with the intuition that the larger the price-wedge for intermediate goods between the two economies, the larger (i.e. bigger) offshoring flows from the developed nation, and the higher the demand for type-h intermediate good, $h = i,j$.

In the Appendix section we show that optimality in the developing country's intermediate good sector is subject to the following first order conditions:

\[
\begin{align*}
(1 - \varepsilon)^{-1} \gamma \theta P_{jt}Y_{jt} &= R_{jt}K_{jt} \quad (22) \\
(1 - \varepsilon)^{-1} (1 - \gamma) \theta P_{jt}Y_{jt} &= W_{jt}H_{jt} \quad (23)
\end{align*}
\]

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12 Using a standard downward-slopping demand curve facing the monopolist, equations (20) – (25) capture an upward shift of the demand and marginal revenue curves. The equilibrium section identifies the condition under which the new optimal level of output is to the right of the original one for the monopolist, hence the fundamental mechanism at play in this framework.
Hence the next lemma:

**Lemma 2**: (a) All else equal, imperfect competition (i.e. low $\theta$) lowers the return to human and physical capital.
(b) All else equal, offshoring activities from abroad raise the return to both human and physical capital.

**Proof**: In the appendix section we show that

\[
R_t = \gamma \theta (Y_t + \rho_t X_t) / (1 - \gamma) K_t
\]

\[
W_t = (1 - \gamma) \theta (Y_t + \rho_t X_t) / (1 - \gamma) H_t
\]

The result follows by way of differentiation with respect to $\epsilon$.
This ends the proof.

Lemma 2 shows that a price-wedge over intermediate goods between the two countries has the potential to trigger upward pressures on factor returns in the developing economy. We expand further on this partial equilibrium result in the equilibrium section.

Furthermore, once combined with optimality in consumption, firms' optimal picks in the light of offshoring activities from abroad may impact factor prices, a possibility that we investigate in the first proposition below. Lemma 3 first sets a connection between factors price and factors availability based on optimality in consumption and production:

**Lemma 3**: The higher the expected stock of physical capital relative to human capital, the higher the current relative price of human capital.

**Proof**: The proof proceeds by substituting equations (26) and (27) into equation (17):

\[
\rho_t = (1 - \gamma) \gamma^{-1} E_t (K_{t+1} / H_{t+1})
\]

This ends the proof.

Lemma 3 illustrates the important point that if offshoring is expected to come with an inflow of capital, say, because of foreign direct investments, then the relative price of human capital might increase in the current period. In fact, equation (28) shows that anticipations of human capital
shortage relative to physical capital will trigger upward pressures on the current relative price of human capital goods. This is because forward-looking agents typically react to such good prospects for the economy by raising their demand for skill-enhancing programs in the current period. This analysis suggests that the potential effects of offshoring on economic growth may be dampened in the event of simultaneous inflows of FDI.

The next section investigates some of the equilibrium effects of offshoring packaging activities when transportation costs outweigh the developing country's price-advantage over intermediate goods.

3.1. Equilibrium Analysis

The following definition introduces the equilibrium concept underlying our study.

Definition (Inter-temporal General Equilibrium): An inter-temporal general equilibrium for this model economy is a sequence of prices, \( \{ \rho_t, \rho_{lt}, P_{lt}, R_{lt}, R_{jt}, R_{xt}, R_{yt}, R_t, W_{lt}, W_{jt}, W_{xt}, W_{yt}, W_t \}_{t=0}^{\infty} \), a sequence of consumption \( \{ C_t \}_{t=0}^{\infty} \), and investment levels \( \{ I_t, X_t \}_{t=0}^{\infty} \), and a sequence of resource allocations across firms, \( \{ K_{ht}, H_{ht} \}_{t=0}^{\infty} \), \( h = i, j \), and sectors \( \{ K_{xt}, K_{yt}, H_{xt}, H_{yt} \}_{t=0}^{\infty} \), such that, for all \( t \):

1. \( W_{lt} = W_{jt} = W_{xt} = W_{yt} = W_t^* \) and \( R_{lt} = R_{jt} = R_{xt} = R_{yt} = R_t^* \);
2. given prices \( \{ \rho_t, \rho_{lt}, P_{lt}, W_t, R_t \} \), \( C_t \), \( H_{t+1} \) and \( K_{t+1} \) solve \( (13) - (16) \), \( I_t = K_{t+1} \) and \( X_t = H_{t+1} \);
3. given total output for the consumption good \( Y_t \), \( K_{xt}, K_{yt}, H_{xt}, H_{yt}, W_t \) and \( R_t \), solve \( (20) - (25) \);
4. all markets clear.

The next definition complements the above one by further specifying the conceptual framework of our study.

Definition (Balanced Growth Path): Along a balanced growth path the ratio of (physical) capital to aggregate output is constant.

The second definition above underlies our only theorem.
Let

\[ \theta \beta < (1 - \varepsilon) \quad (29) \]

where \(0 < \theta < 1\), \(0 < \varepsilon < 1\) and \(0 < \beta < 0\), then:

**Theorem:** There exists a balanced growth path for the developing economy.\(^{13}\)

**Proof:** Using equation (19) it can be shown that \(K_t/\psi_t = B\varepsilon^{-1}(K_t/H_t)^{1-\gamma}\). In the appendix section we also show that \(K_t/H_t = (1 - \gamma) \theta \beta / (1 - \varepsilon)^{-1}\). Thus,

\[ K_t/\psi_t = B\varepsilon^{-1}[(1 - \gamma) \theta \beta (1 - \varepsilon)^{-1}]^{1-\gamma} \quad (30) \]

This ends the proof.

While proving the existence of a balance growth path, equation (30) also shows that offshore activities increase the ratio of physical capital to total output in the developing economy. Thus offshoring from abroad may positively impact the overall capital intensity of the economy.

Since offshoring does not necessarily involve a transfer of resources between developed and developing nations, the notion that it might raise the share of physical capital out of total output is all but trivial. We draw upon several of our results in this paper to rationalize the above finding. Later on in our study, we find that offshore activities are initially welfare reducing, although they foster consumption growth thereafter (see Proposition 4). In the absence of joint capital and offshoring flows, the host country adjusts to the increased demand for capital by initially reducing aggregate consumption (individuals devote more resources to human and physical capital accumulation as shall be shown in proposition 2). Afterwards, physical capital, human capital, aggregate output and consumption all grow at the same rate, as can be seen from proposition 4 and the corollary to proposition 2 below. Thus, the initial change in the host country's capital intensity turns into a permanent feature. On our way to featuring the dynamics of this economy, we first highlight some important results pertaining to factor prices and returns.

**Proposition 1:** Let condition (29) hold. Then: (1) The larger are offshoring inflows, the higher is the return to physical capital.

(2) The larger are offshoring inflows, the larger is the return to human capital and the higher is the relative price of "human capital goods."

\(^{13}\) Unbalanced growth may arise in the event of sector-specific shocks.
Proof: Provided in the appendix section.

The results above still pose a challenge in that inward waves of offshoring raise both the cost of, and the return to, human capital accumulation. Hence it is not clear whether a rational individual would opt for accumulation in such situation. The next proposition clears out this issue.

**Proposition 2**: Let condition (29) hold. Then, in the long run, offshoring fosters both human and physical capital accumulation in the host country.

Proof: Provided in the appendix section.

Proposition 2 is important because it suggests that offshoring may foster economic growth through its fundamentals, *i.e.* through factors accumulation.

Proposition 2 arises as the net benefit of human capital accumulation - which we define in natural logarithmic terms as the difference between, say, the return to human capital accumulation, \( W_t \), and its cost, \( \rho_t \) - increases with the scope of offshoring inflows. In fact, it can be shown that:

\[
\left( \frac{W}{\rho} \right) = (1 - \varepsilon)^{-\gamma} (1 - \gamma)^{(1-\gamma)} \beta^{-\gamma} \psi \theta A.
\]

Similarly, since the consumption good is our *numeraire*, that the net return to physical capital is higher follows directly from Part 1 in Proposition 1 (offshoring raises the direct return to physical capital).

With the above results in hand, a natural extension is to formally rationalize the impact on economic growth in the host country.

**Proposition 2 (Corollary)**: Let condition (29) hold. Then, offshoring fosters economic growth in the host country. The larger the flows, the higher the growth rate of the economy.

Proof: The result follows from (19) and proposition 2, which together imply that \( K, H \) and \( \psi \) growth at the same pace.

This ends the proof.

Proposition 2 and its corollary were derived under the simplifying assumption that production functions are identical for both human capital and tangible intermediate goods. As a robustness check we relax that assumption below as we seek to provide more insight into the growth process
of this small open developing economy. The next proposition speaks to the available evidence according to which developing countries' exports are increasingly becoming capital intensive.

**Proposition 3:** Let condition (29) hold. If in addition output is relatively more responsive to capital in the y-industry compared to the x-industry, then in the long run offshoring raises the capital intensity of tangible goods in the host country.

**Proof:** To prove the result, we consider the following production functions:

\[
Y_{it} = A(K_{it})^\gamma (H_{it})^{1-\gamma}, \quad 0 < \gamma < 1; \quad X_{it} = Z(K_{it})^\alpha (H_{it})^{1-\alpha}, \quad 0 < \alpha < 1.
\]

In the appendix section, while generalizing Proposition 2 we also show that

\[
\kappa_y = (1 - \varepsilon)\gamma/(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta\beta,
\]

where \(\kappa_y \equiv (K_{yt}/K_t)/(H_{yt}/H_t) = (K_{yt}/H_{yt})/(K_t/H_t)\).

Differentiating \(\kappa_y\) with respect to \(\varepsilon\) then yields the result, i.e.

\[
d\kappa_y/d\varepsilon = (\gamma - \alpha)\gamma\theta\beta/(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta\beta)^2.
\]

This ends the proof.

The intuition for this result is as follows. Assume physical capital is more productive if used to produce tangible goods instead of human capital ones, i.e. it impacts output more in the former sector. Then, in order to boost output when experiencing a demand shock, the y-industry emphasizes physical capital relatively more than the x-industry. Capital goods will then become more capital intensive over time, even after controlling for the capital intensity of the whole economy.

Proposition 3 is consistent with recent empirical evidence showing that developing countries' exports are increasingly becoming capital-intensive.\(^{14}\)

At this point our results suggest a potential dilemma for an economy that undergoes such transformations. At issue is the challenge to meet the rising capital intensity of the y-industry while satisfying consumption needs. This issue arises as investment and consumption are competing claims over the y-industry's output. In discussing the welfare implications of the

model we proceed under the maintained and simplifying assumption that $\gamma = \alpha$. The next proposition summarizes our findings.

**Proposition 4**: Let condition (29) hold. Then offshore production activities can be initially welfare-reducing, but foster consumption growth thereafter.

**Proof**: As shown in the Appendix section, the ratio of total consumption to aggregate output is constant and given by the following equation:

$$\frac{C_t}{\psi_t} = 1 - \theta \beta (1 - \varepsilon)^{-1}$$

The first claim then follows by way of differentiation with respect to $\varepsilon$.

The second claim stems from the fact that $C_t/\psi_t$ is constant. This implies that consumption growth keeps pace with economic growth.

This ends the proof.

As offshoring leads the typical household to accumulate more human and physical capital in the host country, proposition 4 points to an initial crowding-out effect on consumption. However, as shown in Proposition 1 offshoring also raises the return to both types of investment, hence allowing the typical household to afford more consumption later on.

So far, we have assumed that the price-wedge between the two countries is not worth transportation costs. This has allowed us to derive some important results within a fairly transparent framework. As long as the developed nation displays more human and physical capital as one should expect, we have shown that a given amount of intermediate input is likely to be cheaper in the developing nation. Assuming a significant price-wedge or negligible transportation costs between the two economies, a firm might find it profitable to carry out product assembly in the developed nation using intermediate physical goods from the developing country. While the analysis would be more sophisticated the main intuition would remain, i.e. differences in factor endowments underlie a price-wedge favorable to the developing country, and this feature in turn leads to a higher demand for intermediate goods. Economic growth ultimately unfolds as individuals find it rewarding to accumulate human and physical capital.\(^{15}\)

\(^{15}\) The developing nation would even experience a higher growth rate if the price differential over the final consumption good is worth transportation costs between both countries. In that case a firm that offshores product assembly to the developing country also sells part of its output in the domestic market, thus boosting local demand for intermediate goods even further.
In the next subsection we study the real terms version of the developing country's net asset position when the final consumption good can profitably be sold in the developed nation by the offshoring firm. Grossman and Rossi-Hansberg (2008) for instance point out that workers in India are reported to be reading x-rays, developing software, preparing tax forms (for U.S. companies), and even performing heart surgery on US patients.\footnote{Grossman and Rossi-Hansberg (2008).}

4. Offshoring Inflows and Final Goods Exports: Implications for the Current Account in the Host Country

Letting domestic and foreign consumption goods be perfect substitutes and endowing consumers with identical preferences internationally, the small open economy assumption implies that export and import prices are determined in world markets. Letting $NX_t$ denote net exports, from the developing country's perspective total expenditures over the consumption good can be expressed as follows:

$$Y_t = C_t + I_t + NX_t,$$  \hspace{1cm} (32)

Since general equilibrium effects can be quite blurry when using an endogenous growth model to investigate open economy issues, we shall restrict attention to the steady state of the economy.

Definition (Steady State): A steady state equilibrium is a general equilibrium which in addition satisfies $K_t^* = K_{t+1}^* = K^*$ and $H_t^* = H_{t+1}^* = H^*$ for all $t$, where $K^*$ and $H^*$ denote the steady-state values of physical and human capital respectively.

Following the lead of Obstfeld and Rogoff (1995), and Sheffrin and Woo (1990), the inter-temporal model of the current account can be expressed in real terms as:

$$CA_t = \Delta F_{t+1} = NX_t + r_tF_t,$$  \hspace{1cm} (33)

where $r_t$ is the real interest rate, and $F_t$ the net asset position. For a small open developing economy the real interest rate is also exogenously determined, i.e. $r_t = \bar{r}$.

Combining (32) and (33) yields the following expression for the current account:

$$CA_t = \Delta F_{t+1} = Y_t - C_t - I_t + \bar{r}F_t.$$

Thus, in real terms, net foreign asset holdings in the steady state amount to:
\[ F^* = -(Y^* - C^* - I^*)/\bar{r}, \quad (34) \]
hence the result.

**Proposition 5**: Let condition (29) hold. If in addition \( B < (1-\gamma)A \), then offshoring improves the host country's net asset position in the steady state of the economy.

**Proof**: While proving Proposition 1 we showed (in the Appendix section) that \( Y_t = (1-(1-\gamma)(1-\varepsilon)^{-1}\theta\beta)A(K_t)^\gamma(H_t)^{1-\gamma}. \) On the other hand, combining equations (19) and (28) using the result that \( K_t/H_t = (1-\gamma)\theta\beta \) yields \( H_t/\psi_t = (1-\varepsilon)^{\gamma}[1-(\gamma)\theta\beta]^{-\gamma}B^{-1}. \) Thus, using equations (31) and (30) along with the steady state feature that \( K_t^* = K_{t+1}^* = K^* \), equation (34) can be re-written as follows:

\[
\frac{F^*}{\psi^*} = \frac{\theta\beta}{(1-\varepsilon)\bar{r}B} \left[(1-\gamma)A - B \left(\frac{1-\varepsilon}{\theta\beta}\right) \gamma(1-\gamma)^{1-\gamma}\right] - \frac{A - B}{\bar{r}B}.
\]

Next, totally differentiating the above with respect to \( \varepsilon \) yields:

\[
\frac{d(F^*/\psi^*)}{d\varepsilon} = \frac{\theta\beta}{(1-\varepsilon)^{2}\bar{r}B} \left[(1-\gamma)A - B \left(\frac{1-\varepsilon}{\theta\beta}\right) \gamma(1-\gamma)^{2-\gamma}\right]
\]

Therefore, it suffices that \( B < (1-\gamma)A \) for \( [d(F^*/\psi^*)]/d\varepsilon > 0 \).

This ends the proof.

Since aggregate TFP, \( B \), is a weighted-average of sectoral TFPs, \( A \) and \( Z \), Proposition 5 states that offshoring benefits the host country's net asset position in the very long run provided that tradeable goods account for less than \( 1-\gamma \) of total output. Since offshoring impacts both sectors, the result suggests that adverse effects may be avoided if the sector of tradeable goods does not overwhelmingly dominates the other sector. A plausible rationale is that increased net exports do not backfire through a huge exchange rate appreciation. In fact, if economic activities are mostly centered around tradeables, currency appreciation could map almost one-to-one into a general price increase, hence eroding the price-wedge that underlies offshoring flows from abroad.

**5. Concluding remarks**

The paper develops an intertemporal general equilibrium model of growth to highlight the effects of offshoring on the accumulation of factor inputs and economic growth. The prospects for offshore product assembly arise as the two trading partners display differing factor endowments.
and the developing nation's price advantage over intermediate goods falls short of transportation costs. The results align well with some recent development experiences, especially those of emerging countries including Mexico, India, Brazil and South Korea.

While the paper offers an optimistic view on the issue, the benefits need to be qualified on the grounds that offshoring may also amplify economic fluctuations in the host country. In fact, Bergin, Feenstra and Hanson (2009) document that industries in Mexico that are associated with U.S. offshoring experience fluctuations in employment that are twice as volatile as the corresponding industries in the U.S.

References


**Appendix**

**Deriving the decision rule of monopolist firms in the intermediate sector**

When confronted with incoming waves of offshoring, firm $j$ in the intermediate good sector picks $P_{jt}$ and $Y_{jt}$ to max $\Pi_j$ subject to

$$
\left\{ \begin{array}{l}
H_{jt} = A^{-1/(\gamma-1)}(K_{jt})^{-\gamma/(\gamma-1)}(Y_{jt})^{1/\gamma} \\
K_{jt} = A^{-1/(\gamma-1)}(H_{jt})^{-\gamma/(\gamma-1)}(Y_{jt})^{1/\gamma}
\end{array} \right.,
$$

(A.1)

where max $\Pi_j = P_{jt}(\bar{Y}_{jt}) - R_{jt}K_{jt} - W_{jt}H_{jt}$, and $\bar{Y}_{jt}$ is the modified demand for type-$j$ physical intermediate good. Substituting (A.1) back into the objective function and taking the derivative with respect to $Y_{jt}$ yield:

$$
[1 + \frac{1}{P_{jt} d\bar{Y}_{jt}/\bar{Y}_{jt} dP_{jt} (\bar{Y}_{jt})}] P_{jt} = A^{\frac{1}{\gamma}}(H_{jt})^{-\frac{1-\gamma}{\gamma}}(Y_{jt})^{\frac{1}{\gamma}}(Y_{jt})^{-1} R_{jt}/\gamma
$$

That is, using $P_{jt} d\bar{Y}_{jt}/\bar{Y}_{jt} dP_{jt} (\bar{Y}_{jt}) = -1/(1 - \theta)$, and arranging terms, $\gamma \theta P_{jt} \bar{Y}_{jt} = R_{jt}K_{jt}$. Or, equivalently, $(1 - \varepsilon)^{-1}(1 - \gamma) \theta P_{jt} Y_{jt} = R_{jt}K_{jt}$, since

$$
\bar{Y}_{jt} \equiv (1 - \varepsilon)^{-1} Y_{jt} = (1 - \varepsilon)^{-1} P_{jt}^{-1/(1-\theta)} Y_{jt}
$$

(A.2)

Similarly it can be shown that $(1 - \varepsilon)^{-1}(1 - \gamma) \theta P_{jt} Y_{jt} = W_{jt}H_{jt}$.

Following the same steps as above for firm $i$ thus yields:

$$(1 - \varepsilon)^{-1}(1 - \gamma) \theta \rho_{it} X_{it} = R_{it}K_{it} \text{ and } (1 - \varepsilon)^{-1}(1 - \gamma) \theta \rho_{it} X_{it} = W_{it}H_{it}$$
Proof of Lemma-1

Using (22) and (24) in the text it follows that

\[ R_{jt} = \gamma \theta P_{jt} / (1 - \varepsilon)K_{jt} = R_{yt} = \gamma \theta Y_t / (1 - \varepsilon)K_{yt} \]

\[ R_{lt} = \gamma \theta \rho_{lt} X_{lt} / (1 - \varepsilon)K_{lt} = R_{xt} = \gamma \theta \rho_{t} X_t / (1 - \varepsilon)K_{xt} \]

In equilibrium, \( R_{yt} = R_{xt} = R_t \). Thus,

\[ R_t = \gamma \theta Y_t / (1 - \varepsilon)K_{yt} = \gamma \theta \rho_t X_t / (1 - \varepsilon)K_{xt} = \gamma \theta (Y_t + \rho_t X_t) / (1 - \varepsilon)K_t \]  
(A.3)

Likewise, using (23) and (25) we have that

\[ W_t = \frac{(1-\gamma)\theta}{(1-\varepsilon)} Y_t / H_{yt} = \frac{(1-\gamma)\theta}{(1-\varepsilon)} \rho_t X_t / H_{xt} = \frac{(1-\gamma)\theta}{(1-\varepsilon)} (Y_t + \rho_t X_t) / H_t \]  
(A.4)

The result then follows by way of differentiation with respect to \( \theta \) and \( \varepsilon \).

Proof of the Theorem

Combining (16) with (26) using \( K_{t+1} = I_t \) yields

\[ I_t / C_t = (1 - \varepsilon)^{-1} \gamma \theta \beta E_t (Y_{t+1} + \rho_{t+1} X_{t+1}) / C_{t+1} \]  
(A.5)

Similarly, combining (15) with (27) using \( H_{t+1} = X_t \) also yields

\[ \rho_t X_t / C_t = (1 - \varepsilon)^{-1} (1 - \gamma) \beta E_t (Y_{t+1} + \rho_{t+1} X_{t+1}) / C_{t+1} \]

Adding up the two equations above yields

\[ (I_t + \rho_t X_t) / C_t = (1 - \varepsilon)^{-1} \theta \beta E_t (Y_{t+1} + \rho_{t+1} X_{t+1}) / C_{t+1} \]

Using \( Y_t = C_t + I_t \), this amounts to

\[ (I_t + \rho_t X_t) / C_t = (1 - \varepsilon)^{-1} \theta \beta E_t (1 + I_{t+1} / C_{t+1} + \rho_{t+1} X_{t+1} / C_{t+1}) \]

\[ = (1 - \varepsilon)^{-1} \theta \beta + (1 - \varepsilon)^{-2} \theta^2 \beta^2 E_t (1 + I_{t+2} / C_{t+2} + \rho_{t+2} X_{t+2} / C_{t+2}) \]
Thus, applying the Law of iterated expectations while letting $\theta \beta < (1 - \varepsilon)$ gives

$$\left( I_t + \rho_t X_t \right)/C_t = \theta \beta/(1 - \varepsilon - \theta \beta), \tag{A.6}$$

which upon combination with (A.5) and $Y_{t+1} = C_{t+1} + I_{t+1}$ yields

$$I_t/C_t = \gamma \theta \beta/(1 - \varepsilon - \theta \beta). \tag{A.7}$$

Thus, from $Y_t = C_t + I_t$, it follows that

$$C_t = (1 - \varepsilon - \theta \beta)Y_t/[1 - \varepsilon - (1 - \gamma) \theta \beta], \tag{A.8}$$

$$I_t = K_{t+1} = \gamma \theta \beta Y_t/[1 - \varepsilon - (1 - \gamma) \theta \beta] \tag{A.9}$$

Next, combining (A.6), (A.7) and (A.8) gives

$$\rho_t X_t/Y_t = (1 - \gamma) \theta \beta/[1 - \varepsilon - (1 - \gamma) \theta \beta] \tag{A.10}$$

Thus, from (A.3) it follows that $K_{xt}/K_t = \rho_t X_t/(Y_t + \rho_t X_t) = (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta$.

Similarly, (A.4) implies that $H_{xt}/H_t = \rho_t X_t/(Y_t + \rho_t X_t)$. Therefore, $K_{xt}/K_t = H_{xt}/H_t$.

Or, equivalently,

$$K_{xt}/H_{xt} = K_t/H_t = (1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta. \tag{A.11}$$

This ends the proof.

**Proof of Proposition 1**

The proof proceeds in two steps.

Since $R_t = \gamma \theta (Y_t + \rho_t X_t)/(1 - \varepsilon)K_t$ - see (26) - using (44) and arranging terms yield:

$$R_t = \gamma \theta(1 + \rho_t X_t/Y_t)/(1 - \varepsilon)K_t/Y_t$$

i.e.
Similarly, since $W_t = \theta(1 - \gamma)(Y_t + \rho_t X_t)/(1 - \varepsilon)H_t$, using (A.10) and arranging terms yield

$$W_t = \frac{\theta(1 - \gamma) Y_t}{[1 - \varepsilon - (1 - \gamma)\theta\beta] H_t}$$  \hspace{1cm} (A.13)

**Step 1: Computing $Y_t/K_t$ and $Y_t/H_t$**

Market clearing for factor inputs imposes that

$$K_{yt}/K_t = H_{yt}/H_t = 1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta\beta$$  \hspace{1cm} (A.14)

which states that the $y$-industry uses a fraction $1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta\beta$ of all of the available resources economy-wide. Therefore the total supply of the $y$-good can be derived from:

$$Y_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta\beta]A(K_t)^\gamma(H_t)^{1-\gamma},$$  \hspace{1cm} (A.15)

or, equivalently, $Y_t/K_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta\beta]A(H_t/K_t)^{1-\gamma}$, which can be rewritten using (A.11) as follows:

$$Y_t/K_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta\beta]A[(1 - \varepsilon)/(1 - \gamma)\theta\beta]^{1-\gamma}$$  \hspace{1cm} (A.16)

(Note that under (29), $(1 - \gamma)\theta\beta/(1 - \varepsilon) < 1$).

Likewise (A.15) carries the implication that $Y_t/H_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta\beta](K_t/H_t)^{\gamma}A$.

Once again, using (A.11), this yields

$$Y_t/H_t = [1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta\beta][(1 - \gamma)\theta\beta/(1 - \varepsilon)]^\gamma A$$  \hspace{1cm} (A.17)

**Step 2: Proving the claims**

**Claim 1:** Offshoring raises the return to physical capital in the host country.
We prove this by combining (A.12) and (A.16) to get that
\[ R_t = (1 - \varepsilon)^{-\gamma}[(1 - \gamma)\beta]^{(1-\gamma)\gamma\theta\gamma}A.\]

The result then follows by way of differentiation with respect to \( \varepsilon \).

**Claim 2:** *Offshoring raises the relative price of human capital goods and the wage rate in the host country.*

Since the relative price of human capital goods is given by \( \rho_t = (1 - \gamma)\gamma^{-1}E_t(K_{t+1}/H_{t+1}) \) - see (28) - we use (A.11) to get \( \rho_t = (1 - \varepsilon)^{-1}(1 - \gamma)^2\gamma^{-1}\theta \beta \). On the other hand, combining (A.13) and (A.17) yields \( W_t = (1 - \varepsilon)^{-1+\gamma}(1 - \gamma)^{1+\gamma}\gamma^{1+\gamma}\beta^{\gamma}A. \) The result follows by differentiating these two expressions with respect to \( \varepsilon \).

This ends the proof.

**Proof of Proposition 2**

First, we rewrite (49) in logarithmic terms as follows:
\[ y_t = a + \gamma k_t + (1 - \gamma)h_t + \ln[1 - (1 - \gamma)(1 - \varepsilon)^{-1}\theta \beta] \tag{A.18} \]

Similarly, (A.11) implies that the total supply of the x-good can be derived from:
\[ X_t = (1 - \gamma)(1 - \varepsilon)^{-1}\theta \beta Z(K_t)^{\gamma}(H_t)^{1-\gamma}, \tag{A.19} \]

*i.e.*, in log terms,
\[ x_t = z + \gamma k_t + (1 - \gamma)h_t + \ln[(1 - \gamma)(1 - \varepsilon)^{-1}\theta \beta] \tag{A.20} \]

Next, combining (A.9) and (A.15) gives the level of accumulated physical capital in the next period (in log terms) as:
\[ k_{t+1} = a + \ln(\gamma \theta \beta) - \ln(1 - \varepsilon) + \gamma k_t + (1 - \gamma)h_t, \tag{A.21} \]

Or, equivalently,
\[ k_{t+1} - k_t = a + \ln[\gamma \theta \beta/(1 - \varepsilon)] - (1 - \gamma)(k_t - h_t). \]  
(A.22)

Similarly, using (A.19) and \( H_{t+1} = X_t \) we find the level of accumulated human capital in the next period (in log terms) as:
\[ h_{t+1} = z + \gamma k_t + (1 - \gamma)h_t + \ln[(1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta], \]  
(A.23)

i.e. \( h_{t+1} - h_t = z + \gamma (k_t - h_t) + \ln[(1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta]. \)  
(A.24)

On the other hand, (A.21) and (A.23) can be combined to get
\[ \gamma k_{t+1} + (1 - \gamma)h_{t+1} = \gamma \ln(\gamma \theta \beta) - \gamma \ln(1 - \varepsilon) + \gamma k_t + \gamma a + (1 - \gamma)z + (1 - \gamma)h_t + (1 - \gamma) \ln[(1 - \gamma)(1 - \varepsilon)^{-1} \theta \beta] \]  
(A.25)

In clear, using (A.23),
\[ k_{t+1} - h_{t+1} = a - z + \ln(\gamma \theta \beta) - \ln(1 - \gamma) \theta \beta]. \]  
(A.26)

Substituting (A.26) back into (A.22) and (A.24) yields
\[ g_k - g_h = a \gamma + (1 - \gamma)z + \ln[(1 - \varepsilon)^{-1} \theta \beta] + \gamma \ln \gamma + (1 - \gamma) \ln(1 - \gamma). \]

where \( g_h \) and \( g_k \) stand for the growth rate of human and physical capital. The result then follows by way of differentiation with respect to \( \varepsilon \).

**Proof of Proposition 3**

In generalizing proposition 2 we consider the following production functions:
\[ Y_{jt} = A(K_{jt})^\gamma (H_{jt})^{1-\gamma}, \quad 0 < \gamma < 1; \]  
(A.27)

\[ X_{lt} = Z(K_{lt})^\alpha (H_{lt})^{1-\alpha}, \quad 0 < \alpha < 1. \]  
(A.28)

We first prove the claim that offshoring raises the capital intensity of tangible goods in the host country.
From the first order conditions for capital, it can be shown that

\[ R_{jt} = \theta Y_t Y_{jt} / (1 - \varepsilon)K_{jt} = R_{yt} = \theta Y_t / (1 - \varepsilon)K_{yt}, \]
\[ R_{lt} = \alpha \rho_l X_{lt} / (1 - \varepsilon)K_{lt} = R_{xt} = \alpha \rho_l X_t / (1 - \varepsilon)K_{xt}. \]

In equilibrium, \( R_{yt} = R_{xt} = R_t \). Thus

\[ R_t = \theta Y_t / (1 - \varepsilon)K_{yt} = \alpha \rho_t X_t / (1 - \varepsilon)K_{xt} = \theta (\gamma Y_t + \alpha \rho_t X_t) / (1 - \varepsilon)K_t. \]  

(A.29)

Similarly, from the first order conditions for human capital, it can be shown that

\[ W_t = \theta (1 - \gamma) Y_t / (1 - \varepsilon)H_{yt} = (1 - \alpha) \theta \rho_t X_t / (1 - \varepsilon)H_{xt} \]
\[ = [(1 - \gamma) Y_t + (1 - \alpha) \rho_t X_t] \theta / (1 - \varepsilon)H_t \]

Combining (A.29) and (16) using \( K_{t+1} = l_t \) yields:

\[ l_t / c_t = \theta (1 - \varepsilon)^{-1} \beta E_t [(\gamma Y_{t+1} + \alpha \rho_{t+1} X_{t+1}) / c_{t+1}], \]

(A.31)

Likewise, combining (A.30) and (15) using \( H_{t+1} = X_t \)

\[ \rho_t X_t / c_t = \theta (1 - \varepsilon)^{-1} \beta E_t [((1 - \gamma) Y_{t+1} + (1 - \alpha) \rho_{t+1} X_{t+1}) / c_{t+1}]. \]

(A.32)

Following the same steps as before yields \((l_t + \rho_t X_t) / c_t = \theta \beta / (1 - \varepsilon - \theta \beta)\). That is,

\[ \rho_t X_t / c_t = [\theta \beta / (1 - \varepsilon - \theta \beta)] - l_t / c_t. \]

(A.33)

Substituting (A.33) back into (A.31) yields

\[ \frac{l_t}{c_t} = (1 - \varepsilon)^{-1} \theta \beta \left[ y + \frac{\alpha \theta \beta}{(1 - \varepsilon - \theta \beta)} \right] + \theta (\gamma - \alpha) (1 - \varepsilon)^{-1} \beta E_t \left( \frac{l_{t+1}}{c_{t+1}} \right) \]
\[ = (1 - \varepsilon)^{-1} \theta \beta \left[ y + \frac{\alpha \theta \beta}{(1 - \varepsilon - \theta \beta)} \right] + [(\gamma - \alpha) (1 - \varepsilon)^{-1} \theta \beta]^2 E_t (l_{t+2} / c_{t+2}) \]
\[ + (\gamma - \alpha) (1 - \varepsilon)^{-1} \theta \beta (1 - \varepsilon)^{-1} \theta \beta [y + \alpha \theta \beta / (1 - \varepsilon - \theta \beta)] \]
Under condition 1, $(\gamma - \alpha)(1 - \varepsilon)^{-1}\theta \beta < 1$ and it can be shown that

$$I_t/C_t = [(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta]/\beta /[(1 - \varepsilon) - (\gamma - \alpha)\theta \beta]$$

(A.34)

Thus, using $Y_t = C_t + I_t$, after minor algebraic manipulations $\rho_t X_t/Y_t = (1 - \gamma)\theta \beta/(1 - \varepsilon - \theta \beta + \alpha \theta \beta)$. Equation (A.29) and market clearing for physical capital then imply that

$$K_{yt}/K_t = \gamma (1 - \varepsilon - \theta \beta + \alpha \theta \beta)/[(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta],$$

(A.35)

so that

$$K_{xt}/K_t = (1 - \gamma)\alpha \theta \beta /[(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta].$$

(A.36)

Similarly, it can be shown that

$$H_{yt}/H_t = 1 - (1 - \alpha) (1 - \varepsilon)^{-1}\theta \beta,$$

(A.37)

and

$$H_{xt}/H_t = (1 - \alpha) (1 - \varepsilon)^{-1}\theta \beta.$$

(A.38)

Thus, capital intensity in the sector of physical goods relative to that of the economy is given by:

$$(K_{yt}/K_t)/(H_{yt}/H_t) \equiv (K_{yt}/H_{yt})/(K_t/H_t) \equiv \kappa_y = (1 - \varepsilon)\gamma /[(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta].$$

Differentiating the above with respect to $\varepsilon$ yields

$$d\kappa_y/d\varepsilon = (\gamma - \alpha)\gamma \theta \beta /[(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta]^2$$

We now turn to showing that $g_k = g_h$.

Given (A.36), (A.35), (A.37) and (A.38) we now have that

$$Y_t = A[(1 - \varepsilon - \theta \beta + \alpha \theta \beta)\gamma K_t /((1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta)]^\gamma [(1 - (1 - \alpha)(1 - \varepsilon)^{-1}\theta \beta)H_t]^{1 - \gamma},$$

$$X_t = Z[(1 - \gamma)\alpha \theta \beta K_t /((1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta)]^{\alpha} [(1 - \alpha)(1 - \varepsilon)^{-1}\theta \beta H_t]^{1 - \alpha}.$$ 

Likewise, combining (A.34) and $Y_t = C_t + I_t$ now yields

$$C_t = (1 - \varepsilon - \theta \beta) [(1 - \varepsilon) - (\gamma - \alpha)\theta \beta] Y_t / (1 - \varepsilon) [(1 - (1 - \alpha)\theta \beta)],$$

and

$$I_t \equiv K_{t+1} = [(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta] Y_t / (1 - \varepsilon) [1 - (1 - \alpha)\theta \beta].$$
As before, the counterparts to (A.22), (A.24) and (A.25) are respectively:

\[
k_{t+1} - k_t = a - (1 - \gamma)(k_t - h_t) + \ln \left[ \frac{\theta \beta}{1 - \varepsilon} \right] + \ln \gamma - \ln[1 - (1 - \alpha)\theta \beta] + (1 - \gamma)\ln[((1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta)/(1 - \varepsilon)] + \ln [1 - \varepsilon - (1 - \alpha)\theta \beta] \tag{A.39}
\]

\[
h_{t+1} - h_t = z + \alpha(k_t - h_t) - (1 - \alpha)\ln(1 - \varepsilon) + \alpha \ln[(1 - \gamma)\alpha \theta \beta] - \alpha \ln[(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta] + (1 - \alpha)\ln(1 - \alpha)\theta \beta \tag{74}
\]

and

\[
\alpha k_{t+1} + (1 - \gamma)h_{t+1} = aa + (1 - \gamma)z + ak_t + (1 - \gamma)h_t + \alpha \ln(\theta \beta) + \gamma \alpha \ln \gamma - \alpha \ln(1 - \varepsilon) + \alpha \ln[1 - (1 - \alpha)\theta \beta] + (1 - \gamma)\alpha \ln[(1 - \gamma)\alpha \theta \beta] + \alpha \ln[1 - \varepsilon - (1 - \alpha)\theta \beta] - (1 - \gamma)\alpha \ln(1 - \varepsilon) + (1 - \gamma)(1 - \alpha)\ln[(1 - \alpha)\theta \beta] - (1 - \alpha)(1 - \gamma)\ln(1 - \varepsilon) \tag{A.40}
\]

We note that (A.40) shows that \(\alpha (k_{t+1} - k_t) + (1 - \gamma)(h_{t+1} - h_t)\) is constant, so that \(g_k = g_h\).

This in turn implies that the right-hand sides in (A.39) and (A.40) are equal and that

\[
(1 + \alpha - \gamma)(k_t - h_t) = a - z + \gamma \alpha \ln \gamma - (1 - \alpha)\ln(1 - \alpha) - \ln[1 - (1 - \alpha)\theta \beta] - (1 + \alpha - \gamma)\ln(1 - \varepsilon) + \ln[1 - \varepsilon - (1 - \alpha)\theta \beta] + (1 + \alpha - \gamma)\ln[(1 - \varepsilon)\gamma - (\gamma - \alpha)\theta \beta] - \alpha \ln[(1 - \gamma)\alpha]
\]

Substituting the above back into (A.39) for \((k_t - h_t)\) and arranging terms finally yields:

\[
g_k \equiv k_{t+1} - k_t = \ln(1 - \varepsilon)^{-1} \theta \beta + \frac{\alpha}{1 + \alpha - \gamma} \ln \left[ \frac{1 - \varepsilon - (1 - \alpha)\theta \beta}{1 - (1 - \alpha)\theta \beta} \right] + \frac{aa + (1 - \gamma)z}{1 + \alpha - \gamma} + [(1 - \gamma)(1 - \alpha)/(1 + \alpha - \gamma)]\ln(1 - \alpha) + [(1 - \gamma)/(1 + \alpha - \gamma)]\ln [(1 - \gamma)\alpha] + [\alpha \gamma/(1 + \alpha - \gamma)]\ln \gamma
\]

**Proof of Proposition 4**

**Claim 1:** Offshoring is initially welfare-reducing.

Combining (18) and (A.6) using \(K_{t+1} = I_t\) gives \(C_t/\psi_t = 1 - \theta \beta (1 - \varepsilon)^{-1}\). Differentiation with respect to \(\varepsilon\) then yields the result.